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MONETARY POLICY, LIQUIDITY STRESS AND LEARNING DYNAMICS

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Monetary Policy, Liquidity Stress and Learning Dynamics*

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Abstract

This paper examines the interactions between monetary policy and stability of interbank money markets. After showing some empirical evidence of a central bank's concern for money market stability I derive a forward smoothing interest rate rule moving from an explicit target in terms of a liquidity stress indicator. The implications of this approach on equilibrium determinacy and learnability are analyzed. I show that equilibrium uniqueness is not necessarily compatible with equilibrium learnability, and learnability, in general, has tighter requirements than determinacy.

JEL classification: E43, E44, E52, E58
Keywords: LIBOR-OIS spread, Taylor Rule, Adaptive Learning, DSGE models, Monetary Policy

1 Introduction

During the phase of higher stress for money market in the 2007-2009 crisis, the LIBOR-OIS spread, that is a measure of costs for insuring against future fluctuations of policy rates, increased from a typical normal value of about 10 b.p. to the peak of 364 b.p. Major central banks engaged to offset the increasing costs of wholesale financing because ultimately the latter would have determined a credit crunch for firms. As far as the Federal Reserve is concerned, the goal of restoring normal market conditions has been pursued through both a vast program of financial assets' term acquisition and a reduction of the operating target. Thus, in order to reduce the LIBOR-OIS spread, the Fed issued new instruments in order to provide funds to institutions that were experiencing some difficulties in getting access to credit on the interbank money market. Furthermore, the Fed induced a decline of the federal funds' rate. The target fed funds value is typically meant to depend on output gap and inflation expectation. Nevertheless, none of these variables in late 2007 could explain the decline of the fed's target. An hypothesis that can help explaining the movement of the target rate with respect to the underlying determinants is that Fed reacted also to some indicator of distress in liquidity market.

In this paper central bank fixes its target taking into account information from the LIBOR-OIS spread, thus responding directly to distress in the interbank money market by inducing a decrease of fed funds' rate. This very simple approach makes possible easy comparisons with standard results in terms of model stability. Curdia and Woodford (2010) produced a normative analysis about the consequences on welfare of adopting

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a similar Taylor rule augmented for a credit market spread. They prove that under some conditions this kind of rule is welfare increasing.

Even though the Fed possibly reacted heavily to turmoils in interbank market liquidity pursuing the macroeconomic stability goal, the majority of the papers about this phenomenon focused on market inefficiencies such as, for instance, counterparty risk (Heider et al., 2009), lack of hedging opportunities (Allen et al., 2009), insufficient interbank liquidity insurance (Castiglionesi and Wagner, 2009). This paper instead has a positive macroeconomic perspective, though policy recommendation in a microeconomic dimension too can be drawn. In a simple DSGE framework it is showed how the policy reaction to liquidity market turmoils can be usefully analyzed by means of a forward smoothing Taylor rule, i.e. a Taylor rule reacting to current prediction on future interest rate. The benchmark analysis with rational expectations is extended by assuming that the private sector forms expectation by using an adaptive learning procedure. In this framework private sector expectations are included into the policy maker information set as a different variable with respect to its own expectations. Following this approach implies that expectations about macro aggregates are essentially driven by expectations about future policies rates, which, in turn, react to private sector expectations about policy. This makes monetary policy sensitive to the expectations' formation process, and, as far as we are concerned, to the learning mechanism and dynamics.

Just like in the case of a backward smoothing rule, the current interest rate sensitivity to its current expectations is a policy constant parameter, whose value is determined as a linear combination of all the policy parameters. Thus, the value of this coefficient is itself a policy parameter, and, as a such, it affects both equilibrium conditions and the stability under learning. In this framework the central bank preferences about stability in liquidity markets are expressed by the magnitude of the policy parameter on the LIBOR-OIS spread. One of the results of this paper is that such preferences may affect economy's stability. Therefore, when fixing a target in terms of interbank markets stability, such consequences should be taken into account in order to avoid equilibria that are not learnable.

This paper provides the analytical derivation of a forward interest rate smoothing moving from the explicit need to react to an index of liquidity stress. In doing so a different approach is followed with respect to previous works focusing on basis risk (see Di Giorgio and Rotondi (2009) for a clear analytical derivation and Driffill et al. (2006) for empirical evidence about Fed's concerns for basis risk). Also it is provided empirical evidence of statistical significance of the LIBOR-OIS spread's influence on policy rate actual evolution. The equilibrium implications in a dynamic stochastic general equilibrium framework are examined, as well as the learning dynamics with agents learning adaptively and behaving like econometricians.

The paper is structured as follows. In Section 2 an interest rate rule that explicitly includes an index of distress in liquidity markets is manipulated in order to be easily encompassed into a DSGE framework. In Section 3 it is provided some empirical evidence of a central bank's concern for liquidity markets' stability. Section 4 examines the dynamics of a basic DSGE model when monetary policy follows the rule derived in section 2. Section 5 summarizes and concludes.

2 The policy rule

This paper aims at analyzing the equilibrium implications of an explicit concern of central bank for stability in the liquidity market stability. Therefore it is assumed that central bank takes into account a liquidity stress index while determining its policy target. An index that is typically used as a stable indicator of money market stability is the LIBOR-OIS spread. LIBOR is an interbank interest rate for unsecured funds. An

Overnight Indexed Swap (OIS) is a fixed/floating interest rate swap. The floating leg (i.e. the component of the net cash flow depending on a floating interest rate) is connected to an overnight interest rate index. Such an index is daily published and is computed as a function of the past values of the overnight interest rate. The observable market OIS rate is the fixed rate in an OIS agreement. Hence, for a given term, the fixed rate cash flow is exchanged for the cash flow determined by a daily published floating index. An overnight indexed swap position replicates the accrual on a notional amount at the index rate, every business day over the term of the swap. In fact the index is generally computed as the weighted average rate for overnight transactions as published by the central bank. Actually, this kind of agreement implies an exchange at maturity of the difference between interest accrued on the agreed notional amount at the agreed fixed rate, and interest accrued through geometric averaging of the floating index rate. Thus, receiving the fixed rate in an OIS agreement is equivalent to lending cash, as paying the fixed rate is equivalent to borrowing cash. In fact, if the fixed rate receiver could borrow cash on the same maturity as the swap, and lend back every day in the market at the index rate, the cash pay-off at maturity would exactly match the OIS swap payout (Dwyer and Tkac, 2009). Thus, the OIS perfectly hedges a cash underlying. In normal market conditions the differences between the LIBOR and the OIS rate should be eliminated by arbitrage (e.g. lending at LIBOR for a given term and borrowing overnight on money market, rolling over the fund until the term expiration), but a spread should keep existing because, due to potential unexpected and undsired fluctuations of the overnight rate during the life of the agreement, the two agreements are not equivalent. The spread between the OIS rate and the LIBOR actually represents the cost of insuring against fluctuation of the floating rate, that indeed is the cost of entering an OIS.

A bank entering into an OIS agreement accepts to receive a fixed rate (the OIS rate) over a notional amount of capital for a given term, while paying a floating compound interest rate based on the overnight interbank rate. This swap agreement allows the bank to get a longer-term interest rate stability in funding with respect to rolling over on a daily basis at changing overnight rates. Therefore, banks may be willing to pay a premium for such a stability. The LIBOR-OIS spread reflects the premium for the immediate availability of a longer term funding, and, as a such, it may reflect the effective liquidity needs of the banking industry. The OIS rate may also be interpreted as the market's expectations about future policy rates. The timing difference between the two rates involved in the OIS agreements implies that the OIS rate represents a market price for insuring against fluctuations in money market rates, that is, the main policy target. The LIBOR-OIS spread represents indeed the market assessment of a generalized measure of credit risk and a measure of capital shortening[1]. Actually, as LIBOR loans are not collateralized, the LIBOR-OIS spread might represent the market assessment of the strength of balance sheets. For a bank in needs of funds the LIBOR-OIS spread represents the shadow price of capital because it reflects the need to face unexpected fluctuations at mismatching maturities. None of these two interpretations is immune to critiques, as, if the former is true and the LIBOR reflects creditworthiness, spreads should vary consistently across banks, but this is not the case, and if the latter is true, existence and persistence of the spread would imply that all banks are capital constrained, or that even few big banks may arbitrage it Giavazzi (2008). There is evidence across the literature Michaud and Upper (2008) Frank and Hesse (2009) that both the component might prevail in turn, according to changing market and system conditions.

For our purposes of monetary policy analysis, we need to include the properties of the LIBOR-OIS spread into an analytically tractable variable, such that it will be used as an argument of the policy reaction function. The analytical framework of our assessments is a discrete-time standard DSGE model. Typically the period

[1] Among other possible interpretations see for instance Giavazzi (2008).

unit of this class of models refers to quarterly observations, thus we need, in a basic model, to make the timing of variables from financial markets homogeneous, because they usually have higher frequencies.
In the swap market the relevant price at which agreements are signed is the fixed interest rate. The bid swap rate represents the fixed rate that the market maker accepts to pay in exchange of a floating inflow of payments. The ask swap rate represents the fixed rate that the market maker accepts to receive in exchange of a floating outflow of payments. Thus, what the market participants observe is essentially a time series of the fixed rate r_{OIS}. However, the swap market's capability to indicate potential liquidity tensions is related, instead, to the floating leg. We can observe that in the OIS market, for each level of the floating interest rate, the bank entering into a OIS should receive a lower fixed rate in order to ensure against fluctuations of the overnight rate. Thus, we can use the inverse of the fixed OIS rate as a target policy maker's variable.
We are going to consider the LIBOR-OIS spread index as the difference between the actual interbank rate and the expectations on its future value. By construction the OIS index embodies expectations about interbank rates. In the OIS market the floating rate is determined as follows:

$$r^{OIS} = \exp\left(\frac{1}{n}\sum_{j=1}^{n}\log\kappa_j^{OIS}\right),$$

where κ_j^{OIS} is an *ad hoc* index referring to the weighted average of overnight transactions in the previous periods, and n is the total number of period considered in the computation of the OIS index. Coherently with the actual determination of such an index, I assume that r^{OIS} represents the period by period expectation over the future policy rate. I consider the OIS rate as the geometric average of the expectations on the prevailing policy rate at the current time, but, in reason of the discrete timing, I allow for two points at which expectations are taken, that is, the beginning and the end of the period. Thus, in the lapse of time between t and $t+1$, I consider the geometric average between the expectations of r_t at time t (i.e. r_t), and the expectations of r_{t+1} at the end o period t, when the actual value of r_{t+1} did not realize yet. It can be expressed as:

$$r_t^{OIS} = \exp\left[\frac{1}{2}\log\left(E_t r_t r_{t+1}\right)\right]. \tag{1}$$

It can be observed that the OIS rate is increasing over both its arguments, i.e. the current and expected target policy rate. Thus, this specification of the OIS rate can in principle be used into a basic DSGE. Nevertheless, in order to encompass the OIS rate it is required that the timings of all variables match. Hence, the variable here used need to be transformed from a daily to a quarterly basis. By this procedure it is used essentially the three months LIBOR time series, taking the current value and expectations over three months. The expression in Equation (1) becomes a geometric average between current LIBOR and expected LIBOR over one quarter.
Vanilla LIBOR swaps reflect generically the markets' expectations of future interest rates, hence this procedure does not affect substantially the analysis based on OIS, and in Figure 1 it is possible to observe a substantial comovement between maturity consistent OIS and LIBOR swap rates. The comovement is quite evident until the sharpening of the financial crisis, in August 2007. In the pre-crisis period the two series show a small and almost constant spread. Then, from August 2007 on, the two time series show very similar patterns but the spread widened, and so the volatility. However, with the exception of the first two quarters of 2009, during the financial stress, the two rates generally move to the same direction, but movements do not have the same magnitude. Indeed, accordingly to the spread changes, which are consistent with the

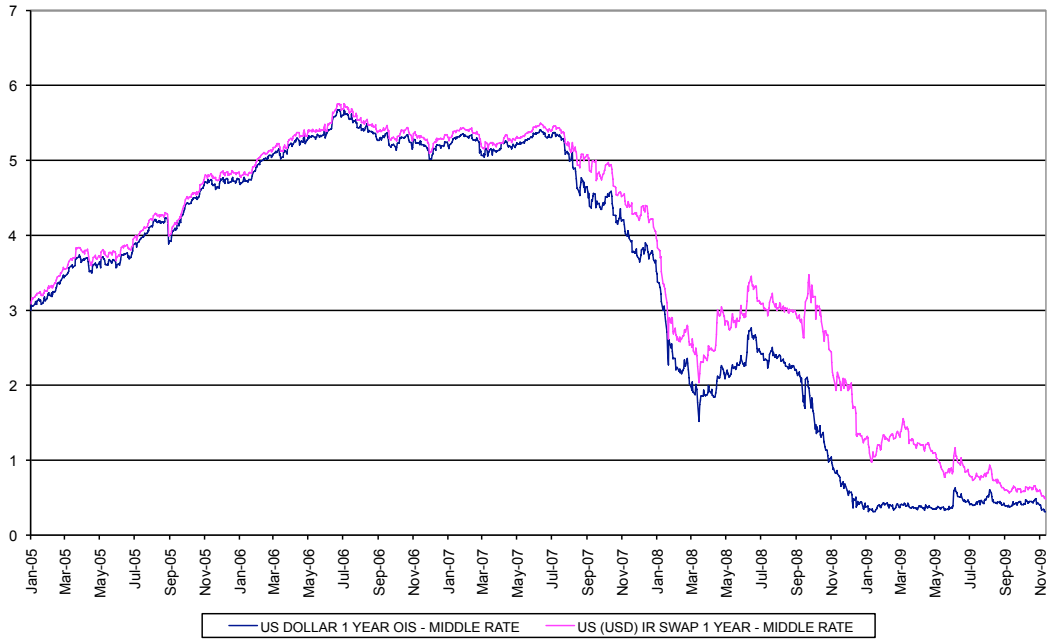


Figure 1: USD Overnight Index Swap rate 1 year, USD Interest Rate Swap 1 year. Source: Datastream

evolution of the financial crisis, I consider two subperiods, one from the origin to august 2007, when the crisis sharpened also due to the bankruptcy filing of American Home Mortgage Investment Corporation, the other from september 2007 to end. In the first subperiod covariance between the two rates is very high and is nearly 0.99. In the second subperiod we can observe a more volatile spread between the two rates. Nonetheless, this does not disruptively affects covariance, which just lowers at 0.96. In the first subperiod the difference between the two rates shows a variance equal to 0.015. In the subperiod after August 2007 the variance of the spread is equal to 0.1225, nearly eight times the previous value.

On the basis of this support and of evidence in Section 3 here it is assumed that central bank reacts to abnormal conditions on the interbank money market. I also assume that central bank sets its target accordingly to the following simple Taylor rule, in which the arguments are inflation forecasts, output gap forecasts and the inverse of the index of liquidity stress:

$$r_t = \phi_1 \pi_{t+1}^{ex} + \phi_2 x_{t+1}^{ex} + \phi_3 \left[LIBOR - OIS \right].$$

After some simple algebra it is possible to reduce the original (and empirical) rule to an analytically tractable version. I approximate the OIS rate by equation (1), then, for ease of calculations I consider the LIBOR-OIS

spread as a percentage difference, thus obtaining the following definition:

$$LIBOR - OIS = \frac{r_t^{OIS}}{r_t} = \sqrt{\frac{r_t^{ex}}{r_t}}.$$

The log-linearization of the Taylor rule $R_t = \Phi\left(\Pi_{t+1}^{ex}, Y_{t+1}^{ex}, (LIBOR - OIS)_t\right)$ gives the following result:

$$r_t = \phi_1 \pi_{t+1}^{ex} + \phi_2 y_{t+1}^{ex} - \phi_3 \log(LIBOR - OIS)_t$$

After some straightforward algebraic manipulation, I obtain a simplified rule in the typical state variables of a basic DSGE model:

$$r_t = \phi_1 \pi_{t+1}^{ex} + \phi_2 y_{t+1}^{ex} + \frac{1}{2}\phi_3 r_t^{ex} + \frac{1}{2}\phi_3 r_t$$

The above relation can be reduced to the following:

$$r_t = \phi_\pi \pi_{t+1}^{ex} + \phi_y y_{t+1}^{ex} + \phi_r r_t^{ex} \tag{2}$$

where

$$\phi_\pi = \frac{2\phi_1}{2+\phi_3} \qquad ; \qquad \phi_y = \frac{2\phi_2}{2+\phi_3} \qquad ; \qquad \phi_r = \frac{\phi_3}{2+\phi_3}$$

It can be observed that the coefficient ϕ_3, representing the policy maker's sensitivity to liquidity market's stability, is inversely related to π_t and y_t. The rule encompasses a forecast on interest rate, whose coefficient is a function of the relative importance that the policy maker puts on the liquidity stress index. This implies that the mechanism of expectations formation might directly affect the monetary policy. This feature proves to be particularly relevant if one examines the learning dynamics, for instance by adopting an adaptive learning procedure.

The reinterpretation of the original empirical rule in a liquidity stress perspective as in equation (2), where interest rate is intended to be set also accordingly to the LIBOR-OIS spread, suggests that this component is not to be considered only during financial turbulence times, but also when normal conditions are generally prevalent, that is when the LIBOR-OIS spread is almost constant. In that case, it will become similar to a component of the constant term, as suggested in Taylor (2008).

3 Empirical evidence

In this section some results are provided in order to support the money markets stabilization motive for Federal Reserve actions. The following econometric estimations aim at assessing the significance of the LIBOR-OIS spread in the process of determination of the target interest rate. The empirical analysis is conducted both for quarterly data and monthly data. In both cases, as remarked in Table 1, Table 2 and Table 3, results suggest a significative role for the LIBOR-OIS spread in affecting the evolution of the fed funds rate. Results and procedures from Clarida et al. (1998) Bernanke and Gertler (1999) for monthly data, and from Clarida et al. (2000) for quarterly data are the benchmarks of the following analysis.

The baseline definition of target policy rate is

$$r_t^* = r^* + \phi_\pi \left(E\left[\pi_{t+1}|\Omega_t\right] - \pi^*\right) + \phi_x E\left[x_{t+1}|\Omega_t\right], \tag{3}$$

where Ω_t represents the information set at time t and r^* denotes the nominal target policy rate in period t as a function of inflation and output gaps from their respective targets (a constant π^* for inflation, potential GDP for output).
In order to allow for interest rate smoothing, the following equation specifies an autoregressive component for the actual Funds rate:

$$r_t = (1 - \mu_1)\, r_t^* + \mu_1 r_{t-1} + \mu_2 r_{t-2} \tag{4}$$

The second order is chosen for better fitting performances. The actual determination of the policy rate combines equations (3) and (4), such that:

$$r_t = (1 - \mu_1 - \mu_2)\,(\alpha + \phi_\pi \pi_{t+n} + \phi_x x_t) - \mu_1 r_{t-1} - \mu_2 r_{t-2} + \varepsilon_t, \tag{5}$$

where n is the forecast horizon. The error term

$$\varepsilon_t \equiv -\,(1 - \mu_1 - \mu_2)\left[\phi_\pi\left(\pi_{t+n} - \pi_{t+n}^{ex}\right) + \phi_x\left(x_t - x_t^{ex}\right)\right]$$

is a linear combination of the forecast errors of inflation and output and, by construction, it is orthogonal to the variables chosen as regressors and instruments. This empirical analysis is extended by adding the LIBOR-OIS spread as a regressor in equation (3). Thus, the complete specification of the actual Funds rate equation that I estimate is the following:

$$r_t = (1 - \mu_1 - \mu_2)\,(\alpha + \phi_\pi \pi_{t+n} + \phi_x x_t + \phi_{LOIS} LOIS) - \mu_1 r_{t-1} - \mu_2 r_{t-2} + \varepsilon_t \tag{6}$$

Estimations of equations (5) and (6) are obtained by Generalized Method of Moments (Hansen, 1982) and particular settings are specified in the respective sections. All the GMM estimations account for HAC weight matrix, whose lags are optimally determined by Bartlett procedure.

3.1 Quarterly data

In this section I followed the methodology of Clarida et al. (2000). Quarterly data are retrieved by St. Louis Fed's database, FRED2, except for LIBOR and OIS rates, for which are used Datastream series. Results from GMM procedures are reported in Table 1, where standard errors are reported in parentheses. In table 1 the first three estimates of equation (5) make use of different subsamples, and are essentially used as benchmarks. The fourth specification refers to equation (6) and is meant to provide support to one of the fundamental statement of this paper, i.e. the importance of LIBOR-OIS spread in determining the monetary policy target. The interest rate that I use is the average of Fed Funds in the first month of the quarter, expressed in annual rates. The inflation rate is the annualized rate of change of the GDP deflator between subsequent quarters. The output gap is the percentage difference between actual GDP and Congressional Budget Office estimates of potential GDP.
The estimated baseline equation is the following:

$$r_t = (1 - \mu_1 - \mu_2)\,(\alpha + \phi_\pi \pi_{t+1} + \phi_x x_t) - \mu_1 r_{t-1} - \mu_2 r_{t-2} + \varepsilon_t$$

where ε has been previously defined. The first estimation uses data in the subsample 1979:3 - 1996:4 in order to compare results with previous literature for the same rule. Results are substantially consistent with the findings in Clarida et al. (2000). The second estimation includes the whole sample. It is used in order to

Table 1: Federal reserve's reaction functions

Sample	ϕ_π	ϕ_x	$\mu_1 + \mu_2$	α	ϕ_{LOIS}	p
Present discounted expected utility	1.95	0.96	0.83	1.97	-	0.99
1979:3 - 2010:3	2.33	1.05	0.88	-0.50	-	0.81
	(0.41)	*(0.29)*	*(0.13)*	*(1.23)*		
2004:1 - 2010-3	1.44	0.77	0.73	2.95	-	0.86
	(0.15)	*(0.15)*	*(0.09)*	*(0.59)*		
2004:1 - 2010:3	0.83	0.06	0.81	2.74	-3.88	1.00
	(0.08)	*(0.02)*	*(0.09)*	*(0.17)*	*(0.27)*	

Quarterly data. The right-most column reports the p-value associated with the Hansen's J statistics, to test the model's overidentifying restrictions. Instruments are lags 3 to 6 of r_t, lags 0 to 3 of π_t, x_t, lags 0 to 4 of commodity price inflation, M2 and spread between long term government bond rate and three-month T-Bill rate. Lags 1 to 4 of *lois* and lags 0 to 2 of *lois* level variation are used as instruments in the fourth specification.

verify the robustness of the previous findings. Results are coherent except for the constant term, that in the whole sample estimation is negative and statistically not-significant. The third and fourth specification use data from 2004:1 to 2010:3, accordingly to the availability of OIS series. The coefficients estimated using this subsample are coherent with the first specification, except for a lower p-value of the overidentification test. Thus, in the 2004:1 - 2010:3 sample we cannot accept nor the alternative hypothesis of a misspecified model, neither the null hypothesis of a correctly specified model. In the fourth specification the LIBOR-OIS (*lois*) regressor is added. The spread is computed as the difference between 3-months dollar OIS rate and the 3-months dollar LIBOR rate. The 3-months maturity is chosen in order to match the model in section 4. Sensitivity to inflation expectations decrease to a level smaller than unity. The output gap, even though still highly significant, has very little influence in determining Fed Funds rate. The constant term is highly significant. The final term, i.e. the LIBOR-OIS spread is highly significant and induces a remarkable improvement in the J-statistics, such that in this case the null of a correctly specified model can be accepted at a significance level smaller than 0.001. The sign of the coefficient is negative, as expected. As the information encompassed by the $LOIS$ regressor is introduced, the expected inflation coefficient decreases. In fact, the informational content in inflation expectations is a subset of that in the LIBOR-OIS spread, as this variable embeds all relevant information about future policy rates.

3.2 Monthly data

In order to test robustness of the general results obtained in the previous subsection, equations (5) and (6) are estimated also on a monthly basis. In this section the methods of Clarida et al. (1998) and Bernanke and Gertler (1999) are applied in order to estimate forward looking reaction functions for the Federal Reserve since 1979. Results from such previous works are essentially confirmed for the sample periods from 1979:10 to 1994:12 and from 1979:10 to 2003:10. The former subsample is used to replicate results from Clarida et al. (1998), the latter extends it until the date of introduction of the OIS rate into a regulated market. Therefore, the analysis is extended by considering the LIBOR-OIS spread. The equation to be estimated is the following:

$$r_t = (1 - \mu_1 - \mu_2)(\alpha + \phi_\pi \pi_{t+12} + \phi_x x_t) - \mu_1 r_{t-1} - \mu_2 r_{t-2} + \varepsilon_t$$

Table 2: Federal Reserve's reaction functions *(output gap 1)*

Sample	ϕ_π	ϕ_x	$\mu_1 + \mu_2$	α	ϕ_{LOIS}
1979:10 - 1994:12	1.40	0.15	0.96	2.71	-
	(0.21)	*(0.078)*	*(0.04)*	*(1.06)*	-
1979:10 - 2010:8	1.98	-0.06	0.97	-0.98	-
	(0.23)	*(0.04)*	*(0.04)*	*(0.72)*	
1979:10 - 2003:10	1.79	-0.05	0.97	0.12	-
	(0.26)	*(0.04)*	*(0.05)*	*(0.93)*	-
2003:10 - 2010:8	1.10	0.12	0.98	-0.21	-
	(0.11)	*(0.03)*	*(0.04)*	*(0.31)*	-
2003:10 - 2010:8	0.46	0.09	0.94	3.29	-3.52
	(0.02)	*(0.00)*	*(0.02)*	*(0.08)*	*(0.11)*

Monthly data. Hansen's J test for overidentifying restrictions shows for all samples p-values higher or equal than 0.999. Hence, the null of a correctly specified model can be accepted at $1‰$ confidence level. The instruments are: 1, x_t, ..., x_{t-6}, x_{t-9}, x_{t-12}, π_t ..., π_{t-6}, π_{t-9}, π_{t-12}, r_{t-2}, ..., r_{t-6}, r_{t-9}, r_{t-12}, lags 1-6, 9, 12 of log-differenced commodity prices, log-differenced CPI and of libor-ois spread (for the last specification).

where α is a constant term, μ_1 and μ_2 are the autoregressive coefficients for the first and the second lag, respectively.

Also in the monthly specification the second order partial adjustment model is adopted in order to make the results comparable to the abovementioned literature, as well as for better fitting reasons. The output-gap considered in Table 2, *outgap1*, is computed as the residual of a regression of the industrial production index on a linear and a quadratic trend. In order to test the robustness of this specification, other measures of output gap are computed. Results of alternative output-gap measures are reported in table 3. The variable *outgap2* is computed as the difference between log-industrial production and the same series filtered by the Hodrick-Prescott algorithm. The variable *outgap3* is computed as the demeaned capacity utilization rate. The results are shown in Table 3 for the complete specification of the rule, that is, the one including LIBOR-OIS spread as an argument (Equation (6)). All the three specification basically imply a significant role for the LIBOR-OIS spread, and its introduction does not affect coefficients for inflation and smoothing. Only the output gap coefficient changes significantly, and this is what it is to be expected due to the changes of the regressor.

All the specifications provide similar results, such that the coefficient ϕ_{LOIS} appears always significant and with the expected negative sign. In fact, as the liquidity stress increases, central bank decreases the policy rate in order to induce some fund easing, and restore normal conditions in the liquidity markets.

4 Equilibrium and learning dynamics

4.1 Determinacy

After having showed that concerns for the liquidity conditions in the money wholesale market may actually affect the policy rates, and that this concern might be represented by an interest-forecast based policy rule,

Table 3: Federal Reserve's reaction functions in the 2003:10 - 2010:8 sample

Output gap measure	ϕ_π	ϕ_x	$\mu_1 + \mu_2$	α	ϕ_{LOIS}
outgap1	0.46	0.09	0.94	3.29	-3.52
	(0.02)	*(0.00)*	*(0.02)*	*(0.08)*	*(0.1082)*
outgap2	0.61	-0.17	0.96	3.86	-5.83
	(0.03)	*(0.01)*	*(0.02)*	*(0.09)*	*(0.11)*
outgap3	0.46	0.10	0.94	3.66	-3.40
	(0.01)	*(0.00)*	*(0.02)*	*(0.06)*	*(0.10)*

Monthly data. Hansen's J test for overidentifying restrictions shows for all samples p-values approximately equal to 1. Hence, the null of a correctly specified model can be accepted at $1^0/_{00}$ confidence level. The instruments are: 1, x_t, ..., x_{t-6}, x_{t-9}, x_{t-12}, π_t ..., π_{t-6}, π_{t-9}, π_{t-12}, r_{t-2}, ..., r_{t-6}, r_{t-9}, r_{t-12}, lags 1-6, 9, 12 of log-differenced commodity prices, log-differenced CPI and of libor-ois spread.

the equilibrium implications are examined in a standard new-keynesian DSGE model:

$$x_t = x_{t+1}^{ex} - \sigma\left(r_t - r_t^n - \pi_{t+1}^{ex}\right) \tag{7}$$

$$\pi_t = \lambda x_t + \beta \pi_{t+1}^{ex} \tag{8}$$

where r_t^n is an exogenous stochastic term representing the natural rate of interest and which is assumed to follow an $AR(1)$ process, such as

$$r_t^n = \rho r_{t-1}^n + \epsilon_t$$

Equation (7) is a dynamic IS derived by log-linearization of the Euler's condition in a representative agent's utility maximization problem. Equation (8) is a Phillips curve derived from optimal pricing decision of firms producing the same final good[2]. The model is closed by the policy rule (2). The model can be expressed in the following state space form:

$$y_{t+1}^{ex} = B y_t + \omega r_t^n \tag{9}$$

where $y_t = (\pi_t, x_t, r_t)'$, $\omega = (0, \sigma, -\sigma\phi_x\phi_r^{-1}, 0)'$ and

$$B = \begin{pmatrix} \beta^{-1} & -\lambda\beta^{-1} & 0 \\ -\sigma\beta^{-1} & 1 + \lambda\sigma\beta & \sigma \\ \dfrac{\sigma\phi_x - \phi_\pi}{\beta\phi_r} & \dfrac{-\beta\phi_x - \lambda\left(\sigma\phi_x - \phi_\pi\right)}{\beta\phi_r} & \dfrac{1 - \phi_x\sigma}{\phi_r} \end{pmatrix}$$

The model (2) (7) (8) has three non predetermined state variables. Therefore, according to the solution proposed by Blanchard and Kahn (1980), for this model to be determinate matrix B must have all three eigenvalues outside the unit circle. Condition for determinacy in a 3×3 model have been worked out in Di Giorgio and Rotondi (2009). The determinant of matrix B is equal to $1/\beta\phi_r$, implying that

$$A_0 = \frac{-1}{\beta\phi_r} < 0.$$

[2]More details about the derivation of this model can be found, among other, in Woodford (2003) and Walsh (2003)

Hence, I refer to Case 2 (Di Giorgio and Rotondi, 2009), such that sufficient conditions for determinacy are the ones stated below:

$$A_0 < 0 \tag{10}$$

$$1 + A_2 + A_1 + A_0 < 0 \tag{11}$$

$$-1 + A_2 - A_1 + A_0 < 0 \tag{12}$$

$$\left|\frac{1}{A_0}\right| < 1 \tag{13}$$

$$A_0^2 - A_0 A_2 + A_1 - 1 > 0 \tag{14}$$

where

$$A_1 = \frac{1 + \beta + \lambda\sigma + \phi_r - \sigma\phi_x - \lambda\sigma\phi_\pi}{\beta\phi_r} \tag{15}$$

$$A_2 = \frac{\beta\sigma\phi_x - (1 + \beta + \lambda\sigma)\,\phi_r - \beta}{\beta\phi_r}. \tag{16}$$

By substitution in conditions (10) and (13), it follows that determinacy requires $\beta\phi_r > 0$. Condition (11) requires that

$$\phi_\pi + \left(\frac{1-\beta}{\lambda}\right)\phi_x > 1 - \phi_r \tag{17}$$

Condition (12) requires that

$$\phi_\pi + \frac{1+\beta}{\lambda}\phi_x < (1 + \phi_r)\left[\frac{2\,(1+\beta) + \lambda\sigma}{\lambda\sigma}\right] \tag{18}$$

Conditions (17) and (18) can be interpreted as a modified version of the Taylor principle with interest forecast based Taylor rule, where the former produces an upper bound for the strength of the reaction to the typical arguments of such a rule, i.e. with inflation and output gap forecasts, and the latter a lower bound, both necessary to avoid indeterminacy.
The last condition that must hold in order to obtain determinacy is (14). It requires, for $\beta > 0$ and $\phi_r > 0$ that the following relation is satisfied:

$$1 - \beta + \beta(1-\beta)\phi_r + (\beta^2 - 1 - \lambda\sigma + \beta\lambda\sigma - \beta\lambda\sigma\phi_\pi - \beta\sigma\phi_x)\phi_r^2 > 0 \tag{19}$$

As a parametric benchmark, it can be observed that condition (19) implies, for $\phi_r = 0$ and $\sigma > 0$ that

$$\phi_x > \frac{\beta - 1}{\beta\sigma}$$

For $\beta \in (0, 1)$ the relationship above requires that ϕ_x be always positive. In practice, typical calibrated values for parameter β and σ satisfy the requirement of their positivity.

4.2 Learning

The analysis of the learning properties of the model refer to the existing literature on adaptive learning, especially to Evans and Honkapohja (2001) and Bullard and Mitra (2002). Here it is assumed that agents

do not have rational expectations. Instead they are supposed to behave homogeneously focusing on some guessed reduced form of the model. This arbitrarily assumed reduced form does not necessarily coincide with the true minimum state variable solution (McCallum, 1983), that represents instead the solution for the rational expectations equilibrium. Thus, agents need to learn the true values of the coefficients they use in their autoregressive guess and here it is assumed that they do this adaptively. Adaptive learning, as it has been developed in Evans and Honkapohja (2001), consists in recursive econometric computations of the relevant parameters. It is assumed that in each period agents act as econometricians and estimate such parameters by ordinary least squares.[3] The learning process by OLS is nested into the economy's dynamics by means of its recursive representation, i.e. by recursive least squares. Agents use the resulting estimations as a basis for taking their real decisions, therefore the economy, by means of the true reduced form, reacts to these potentially biased estimates. The interaction between agents decisions (taken on an arbitrary basis) and the true economy is described in the mapping of each estimated parameter towards its true value. Learning is effective (i.e. estimated parameters are convergent to the true values, identified as the minimum state variable solution) if the E-Stability condition holds. E-Stability is a criterion that governs asymptotic local stability of the parameters mapping around their REE values.
The model (2) (7) (8) can be represented in the form

$$y_t = Ay_{t+1}^{ex} + mr_t^n$$

where $m = (\lambda\sigma, \sigma, 0)'$ and

$$A = \begin{pmatrix} \beta + \lambda\sigma - \lambda\sigma\phi_\pi & \lambda - \lambda\sigma\phi_x & \lambda\sigma\phi_r \\ \sigma - \sigma\phi_\pi & 1 - \sigma\phi_x & -\sigma\phi_r \\ \phi_\pi & \phi_x & \phi_r \end{pmatrix}$$

The minimum state variable solution in this case has the following form:

$$y_t = \bar{a} + \bar{c}r_t^n,$$

where the values of $\bar{a}$ and $\bar{c}$ can be retrieved by undetermined coefficients method and have to be learnt by agents. We can also assume that the reduced form arbitrarily chosen by agents (also known as the perceived law of motion) be structured in a similar fashion. Hence, agents are assumed to correctly identify the state variables of the economy, and to use their perception to formulate expectations, such that:

$$Ey_{t+1} = a + c\rho r_t^n.$$

By inserting these expectations into the actual determination of the model (9) (that represents the true economy reacting to these non perfectly rational economic decisions) we get the actual law of motion of the economy:

$$y_t = aA + (c\rho A + \omega)\, r_n^t$$

Once agents took their decisions, the economy moves accordingly. This process is called Actual Law of Motion (ALM), in contrast to the Perceived Law of Motion (PLM), which is the agents' guess about the reduced form of the model. The dynamics of the estimated parameters, from the arbitrary estimation in

[3] Alternatively, Evans and Honkapohja (2001) propose bayesian learning or constant gain learning.

the current period to the estimation in the following period, affect actual values of the state variables that evolve accordingly. In this sense, parameters can possibly converge to their theoretical REE value if they lead the state variables to values that allow correct estimations. The parameters' dynamics is captured by the following mapping, whose parameters are identified by use of the undetermined coefficients method:

$$T(a, c) = (aA, c\rho A + \omega).$$

The convergence of these map to $T(\bar{a}, \bar{c})$, that is, to its evaluation in correspondence to the parameter values at the rational expectations equilibrium, is represented as the local asymptotical stability of the following matrix differential equation:

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c)$$

The local asymptotic stability of this equation is governed by the E-Stability conditions in Proposition 10.3 in Evans and Honkapohja (2001). In our case the proposition mentioned above reduces to the following statement. Given the equations

$$DT_a(\bar{a}) \equiv A \tag{20}$$

$$DT_c(\bar{c}) \equiv \rho A \tag{21}$$

whose complete derivation can be found in the original work by Evans and Honkapohja (2001), a minimum state variable solution $(\bar{a}, \bar{c})$ is E-stable if all eigenvalues of A and ρA have real part less than 1.
Given that ρ is a real scalar $\in (0, 1)$, E-stability condition on (21) is satisfied if and only if it is satisfied in (20). Therefore, in order for our model to satisfy the E-stability condition, sufficient conditions are represented by the following equations about the sign of the eigenvalues of matrix A:

$$P(1) = 1 + A_2 + A_1 + A_0 > 0 \tag{22}$$

$$A_2 > -2 \tag{23}$$

The proof of the conditions for E-stability comes from the exam of the roots of the characteristic polynomial for matrix A:

$$P(\mu) = \mu^3 + A_2\mu^2 + A_1\mu + A_0 = 0 \tag{24}$$

In order to have all real roots less than 1, the sufficient and necessary conditions are $P(1) > 0$ and $P''(1) > 0$. The first condition is true because the coefficient for the third degree term is unitary, thus, after the largest root, the function $P(\mu)$ is necessarily larger than zero. The second condition is required because for certain values of the coefficients A_2 and A_1, if all roots are real, the function might be decreasing and eventually negative between the smallest and the largest root, even after having reached the positive territory. Thus, we need that the largest admitted real root be on a convex part of the function.
It is known that

$$P''(\mu) = 6\mu + 2A_2$$

which, when evaluated at the largest admitted root, allows to determine a condition for such a function to be strictly smaller. Indeed, if the function has three real roots, it is convex and positive only if it does not intersects the horizontal axis on the right hand side. Therefore:

$$P''(1) = 6 + 2A_2 > 0 \qquad \Longleftrightarrow \qquad A_2 > -3 \tag{25}$$

I need to consider the case of the polynomial (24) having three roots, of which a pair can be a complex pair. If this is the case, conditions (22) and (23) are necessary. In order to be sufficient we need

$$A_2 > -2 \tag{26}$$

Condition (26) encompasses condition (25). In this case (22) ensures that the real root is less than one. In fact, the real root is the single point in the domain where the function intersects the horizontal axis, and we know that if the function has only one real root, given the positive (and unitary) coefficient for the third degree term μ^3 it is increasing around it. Moreover, we know that the matrix A has a positive determinant equal to $\beta\phi_r > 0$. This implies that at least one real root of its characteristic polynomial (24) is larger than zero. If there are two complex roots and one real, condition (26), together with (22), ensures that the real part of the complex pair is less than one. In fact, having imposed the conditions for the real root to be between zero and one, given that the two remaining complex roots are such that $\mu_1 = \alpha + \beta i$ and $\mu_2 = \alpha - \beta i$ we can observe that:

$$\mu_1 + \mu_2 = \alpha + \beta i + \alpha - \beta i = 2\alpha$$

It is known that:

$$A_2 > -2 \quad \Rightarrow \quad -(\mu_1 + \mu_2 + \mu_3) > -2 \quad \Rightarrow \quad \mu_1 + \mu_2 + \mu_3 < 2,$$

$$\mu_1 + \mu_2 + \mu_3 < 2 \quad \Rightarrow \quad 2\alpha + \mu_3 < 2 \quad \Rightarrow \quad \alpha < 1 - \frac{\mu_3}{2},$$

where $\mu_3 \in [0, 1)$, ensuring that $\alpha < 1$.

Condition (22) implies that

$$\phi_\pi + \frac{1-\beta}{\lambda}\phi_x > 1 - \phi_r \tag{27}$$

which is exactly the same condition required (17) for determinacy. Condition (26) implies that

$$\phi_\pi + \frac{\phi_x}{\lambda} > \frac{-1 + \beta + \lambda\sigma}{\lambda\sigma} + \frac{\phi_r}{\lambda\sigma} \tag{28}$$

The right hand side member is larger in (28) than in (27), and smaller than in (18). However left hand side member too in (28) is larger than in (27), and smaller than in (18), therefore we can state that E-stability conditions may imply different policy recommendations, because the values of reaction coefficients might lay in different regions for equilibrium uniqueness and learnability. Moreover, an upper bound to the policy reaction does not exist for learnability of equilibrium.

5 Conclusions

I have analyzed the implications of a Taylor rule augmented by a LIBOR-OIS spread argument. The analysis is conducted in simple DSGE model both under the assumption of rational expectations and of adaptive learning. It is also proposed an econometric assessment of the importance of the LIBOR-OIS spread into the definition of the policy target interest rate. A rational expectations approach is formerly adopted in order to compare the results to the standard results in literature (Clarida et al. (1999), Woodford (2003), Di Giorgio and Rotondi (2009)). Then, the rational expectations hypothesis is dropped out and it is assumed that agents do not have a perfect knowledge of the economy structure, such that they need to estimate in each period the coefficients of the model. We can observe that E-stability conditions, that actually drive the adaptive learnability, are compatible with conditions for determinacy. However they do not result in the same policy prescriptions, i.e. limits on policy parameters are not the same. This implies that a determinate equilibrium may be not learnable. Implementing an interest-forecast based rule may imply a trade off between determinacy and learnability. In abnormal market conditions, i.e. when the required reaction to liquidity market's turmoils, a trade-off between macroeconomic stability and money market stability may arise.

The result of this paper is consistent with a zero lower bound scenario, where Federal Reserve should raise inflation expectations without affecting current real interest rates, in orderto stimulate economy from the monetary side (Walsh, 2009). If too much attention is put on interest rate expectations (i.e. the coefficient ϕ_r is too high) the learnability conditions become heavier, leaving the economy exposed to the danger of sunspot equilibria due to coordination failure (Evans and Honkapohja, 2001). The fact that only future real interest rates can act as a stimulus for aggregate demand in a zero lower bound scenario clearly is not compatible with announcements of low future inflation when the policy interest rate has to stay at zero level. Whenever Fed is engaged in pursuing a cycle stabilization objective while facing a zero lower bound constraint, it also has a constraint given by the influence of current policy rate expectations on stability under learning. As the model presented in this paper focuses on the learning dynamics associated with reaction to expectations, it provides an explanation of the trade off between monetary stimulus in a liquidity trap and lower future inflation, that is a reasonable point if nominal rates reached the zero lower bound. It is showed that a forward looking interest rate rule may not allow learning if expected future interest rate is one of its arguments.

Whenever the Taylor rule includes an explicit relation to interest rate's forecasts, then learnability is not necessarily achieved if central bank looks only at determinacy. The zero lower bound constraint that follows the Fed's non conventional massive monetary policies in response to liquidity stress may produce equilibria agents do not converge to. Therefore, a trade-off between macroeconomic stability and money market stability arises. Misperception of such a trade-off may undermine the dynamic stability of the economy. If policy recommendations do not correctly identify the implicit cost of pursuing interbank market stability, an unseasonable exit strategy may have disruptive consequences in term of macroeconomic stability. Agents' learning process could fail to coordinate expectations, as they perceive that central bank puts too much weight on future interest rate expectations. Thus, endogenous variables could start evolving as sunspots. Even though determinacy is achieved, learnability may be not. Such a structural uncertainty suggests that central bank should adopt a more conservative and cautious behavior in pursuing interbank market stability. Macroeconomic stability should be pursued by means of a preferred target in terms of prices stability. Financial stability, and more in the detail, interbank market stability, should be attained by proper market and intermediaries regulation. Liquidity stress and interbank market troubles in general should be

interpreted as results of market inefficiencies and handled by means of microeconomic tools. Otherwise macroeconomic stability would be under threat, as the correct identification of the parameters determining the E-stability conditions hinges on assumptions about agents' learning procedure. Such assumptions are not clearly identifiable so far, nor they are verifiable. Therefore, caution is crucial in facing interbank market stress, because adaptive learning implies a trade-off between learnability and macroeconomic stability. Nevertheless, welfare consequences of such a trade off are beyond the scope of this work, and they should be analyzed more carefully in specifically calibrated frameworks in order to define more reliable policy recommendations.

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